

SPHERICAL FUNCTORS ON THE KUMMER SURFACE

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ABSTRACT. We find two natural spherical functors associated to the Kummer surface and analyse how their induced twists fit with Bridgeland’s conjecture on the derived autoequivalence group of a complex algebraic K3 surface.

1. INTRODUCTION

Let $\mathcal{D}(X)$ be the bounded derived category of coherent sheaves on a smooth complex projective variety X and $\text{Aut}(\mathcal{D}(X))$ denote the set of isomorphism classes of exact \mathbb{C} -linear autoequivalences of $\mathcal{D}(X)$. Then we always have a subgroup $\text{Aut}_{\text{st}}(\mathcal{D}(X)) \subset \text{Aut}(\mathcal{D}(X))$ of *standard* autoequivalences which is generated by push forwards along automorphisms, twists by line bundles and shifts. The complement of this subgroup, if non-empty, is usually very interesting and mysterious; its elements will be called *non-standard* autoequivalences.

The most successful way to construct non-standard autoequivalences was discovered in the groundbreaking work of Seidel and Thomas [ST01] on *spherical objects*. This was extended by Huybrechts and Thomas [HT06] to a notion of \mathbb{P} -objects and further still, to a theory of *spherical* and \mathbb{P} -functors; see [Rou06, Ann08, Add11].

The first example of a series of \mathbb{P} -functors was constructed by Addington in [Add11, Theorem 2] for the Hilbert scheme $X^{[n]}$ of n points on a K3 surface X . In particular, he showed that the natural functor $F : \mathcal{D}(X) \rightarrow \mathcal{D}(X^{[n]})$ induced by the universal ideal sheaf on $X \times X^{[n]}$ is a \mathbb{P}^{n-1} -functor in the sense of [Add11, §3] and thus gives rise to a non-standard autoequivalence of $\mathcal{D}(X^{[n]})$ for each $n \geq 2$. Notice that when $n = 1$, this F is Mukai’s reflection functor [Muk87, p.362] which coincides (up to a shift) with the spherical twist around the structure sheaf \mathcal{O}_X .

Inspired by this example, the second author [Mea12, Theorem 4.1] provided the analogous result for the generalised Kummer variety $K_n \subset A^{[n+1]}$ associated to an abelian surface A . More precisely, he proved that the natural Fourier-Mukai functor $F_K : \mathcal{D}(A) \rightarrow \mathcal{D}(K_n)$ induced by the universal ideal sheaf on $A \times K_n$ is again a \mathbb{P}^{n-1} -functor yielding a new non-standard autoequivalence of $\mathcal{D}(K_n)$ for each $n \geq 2$.

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This short note completes this theorem to the case $n = 1$ where the generalised Kummer variety is the classical Kummer surface. The motivation to understand this particular case comes from Bridgeland's conjecture [Bri08, Conjecture 1.2] on the derived autoequivalence group of a complex algebraic K3 surface; roughly speaking, it says that $\text{Aut}(\mathcal{D}(X))$ should be generated by standard autoequivalences and twists around spherical objects.

Summary of main results. Every abelian surface A has a natural K3 surface associated to it; namely the *Kummer surface* $K := K_1$. It can either be defined as the blow up of the quotient A/ι along the sixteen ordinary double points, where ι denotes the involution $a \mapsto -a$, or equivalently as the fibre of the Albanese map $m : A^{[2]} \rightarrow A$ over zero. That is, we can identify K with the subvariety of the Hilbert scheme $A^{[2]}$ consisting of those points representing length 2 subschemes of A whose weighted support sums to zero. In other words, there is a universal family $\mathcal{Z} \subset A \times K$ giving rise to the commutative diagram

$$\begin{array}{ccc} & \mathcal{Z} & \\ p \swarrow & & \searrow q \\ A & & K \\ \pi \searrow & & \swarrow \mu \\ & A/\iota & \end{array}$$

Recall that a Fourier-Mukai functor $F : \mathcal{D}(Y) \rightarrow \mathcal{D}(X)$ with left adjoint L and right adjoint R is said to be *spherical* if the cotwist $C_F := \text{cone}(\text{id} \xrightarrow{\eta} RF)$ is an autoequivalence of $\mathcal{D}(Y)$ and we have a functorial isomorphism $R \simeq CL$. In particular, if F is spherical then the *twist* $T_F := \text{cone}(FR \xrightarrow{\epsilon} \text{id})$ is an autoequivalence of $\mathcal{D}(X)$. A spherical object $\mathcal{E} \in \mathcal{D}(X)$ corresponds to the case $F := (_) \otimes \mathcal{E} : \mathcal{D}(\text{pt}) \rightarrow \mathcal{D}(X)$.

In this article, we focus on the exact triangle $F \rightarrow F' \rightarrow F''$ of Fourier-Mukai functors $\Phi_{\mathcal{E}} : \mathcal{D}(A) \rightarrow \mathcal{D}(K)$ induced by the structure sequence of \mathcal{Z} :

$$F := \Phi_{\mathcal{I}_{\mathcal{Z}}} \quad F' := \Phi_{\mathcal{O}_{A \times K}} = H^*(_) \otimes \mathcal{O}_K \quad F'' := \Phi_{\mathcal{O}_{\mathcal{Z}}} = q_* p^*.$$

Our main result is the following

Theorem (2.1 and 2.4). *Both F and F'' are spherical functors with cotwists $C_F \simeq C_{F''} \simeq \iota^*$.*

In light of [Bri08, Conjecture 1.2], this immediately raises the question whether the twists $T_F, T_{F''} \in \text{Aut}(\mathcal{D}(K))$ associated to these *functors* F, F'' can be decomposed into twists $T_{\mathcal{E}}$ around spherical *objects* $\mathcal{E} \in \mathcal{D}(K)$. We answer this question with the following

Theorem (2.1 and 2.4). *The induced twists $T_F, T_{F''} \in \text{Aut}(\mathcal{D}(K))$ decompose in the following way:*

$$T_{F''} \simeq \prod_i T_{\mathcal{O}_{E_i}(-1)}^{-1} \circ M_{\mathcal{O}_K(E/2)}[1] \simeq \prod_i T_{\mathcal{O}_{E_i}} \circ M_{\mathcal{O}_K(-E/2)}[1]$$

and

$$F[1] \simeq T_{\mathcal{O}_K} \circ F'' \implies T_F \simeq T_{\mathcal{O}_K} \circ T_{F''} \circ T_{\mathcal{O}_K}^{-1}$$

where $E = \bigcup_i E_i$ for the exceptional curves E_i of the Hilbert-Chow morphism μ and $M_{\mathcal{O}_K(E/2)} := (_)\otimes \mathcal{O}_K(E/2)$.

It is easy to see that the squares $T_F^2, T_{F''}^2$ of our twists act trivially on the cohomology of K (see [Add11, §1.4]). In fact, Corollary 2.5 shows that $T_F^2 \simeq T_{F''}^2 \simeq [2]$.

In this paper, we will give a different proof of Theorem 2.4 to that which could have been obtained from adapting the arguments in [Mea12]. The advantage of our approach is that it immediately provides us with the decompositions of T_F and $T_{F''}$ as stated above.

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2. NATURAL FUNCTORS ON THE KUMMER SURFACE

Another way of describing K is by first blowing-up the fixed points $\tilde{A} \rightarrow A$. Since the fixed points are ι -invariant, the involution ι lifts to an involution $\tilde{\iota}$ of \tilde{A} .

$$\begin{array}{ccc} & \tilde{A} & \\ p \swarrow & & \searrow q \\ A & & K \\ \pi \searrow & & \swarrow \mu \\ & A/\iota & \end{array}$$

The quotient $\tilde{A} \rightarrow K$ is a double cover ramified over sixteen exceptional curves E_i . Moreover, the canonical bundle formula for the blow-up yields $\omega_{\tilde{A}} \simeq \mathcal{O}(\sum \tilde{E}_i)$ where the \tilde{E}_i are the exceptional divisors in \tilde{A} . Their images E_i in K satisfy $q^*\mathcal{O}(E_i) \simeq \mathcal{O}(2\tilde{E}_i)$ and $q_*\mathcal{O}_{\tilde{A}} \simeq \mathcal{O}_K \oplus \mathcal{O}(-\frac{1}{2}\sum E_i)$. See [Huy14, Chapter 1.1] for more details. We set $E := \bigcup_i E_i$ and $\tilde{E} := \bigcup_i \tilde{E}_i$ from now on.

Proposition 2.1. $F'' : \mathcal{D}(A) \rightarrow \mathcal{D}(K)$ is a spherical functor with cotwist $C_{F''} \simeq \iota^*$ and twist

$$T_{F''} \simeq \prod_i T_{\mathcal{O}_{E_i}(-1)}^{-1} \circ M_{\mathcal{O}_K(E/2)}[1].$$

Proof. Pushforward along the double cover $q_* : \mathcal{D}(\tilde{A}) \rightarrow \mathcal{D}(K)$ is a spherical functor with cotwist $C_{q_*} \simeq M_{\mathcal{O}_{\tilde{A}(\tilde{E})}} \circ \tilde{\iota}^* \simeq S_{\tilde{A}} \circ \tilde{\iota}^*[-2]$ and twist $T_{q_*} \simeq M_{\mathcal{O}_K(E/2)}[1]$; see [Add11, §1.2, Examples 5 & 6].

By [Orl92, Theorem 4.3], we have a semi-orthogonal decomposition

$$\mathcal{D}(\tilde{A}) \simeq \langle \mathcal{O}_{\tilde{E}_1}(-1), \dots, \mathcal{O}_{\tilde{E}_{16}}(-1), p^*\mathcal{D}(A) \rangle$$

We set $\mathcal{A} := \langle \mathcal{O}_{\tilde{E}_1}(-1), \dots, \mathcal{O}_{\tilde{E}_{16}}(-1) \rangle$ and $\mathcal{B} := p^*\mathcal{D}(A)$ so that $\mathcal{D}(\tilde{A}) \simeq \langle \mathcal{A}, \mathcal{B} \rangle$. Since $\mathcal{D}(\tilde{A}) \simeq \langle S_{\tilde{A}}\mathcal{B}, \mathcal{A} \rangle$ by [BK89] and $C_{q_*}\mathcal{B} \simeq S_{\tilde{A}}\mathcal{B}$, we have $\mathcal{D}(\tilde{A}) \simeq \langle C_{q_*}\mathcal{B}, \mathcal{A} \rangle$. Thus, by [HLS13, Theorem 4.13], the restrictions $q_*|_{\mathcal{A}} : \mathcal{D}(A[2]) \rightarrow \mathcal{D}(K)$ (to the set $A[2] \subset A$ of 2-torsion points) and $q_*|_{\mathcal{B}} \simeq q_*p^* =: F'' : \mathcal{D}(A) \rightarrow \mathcal{D}(K)$ are spherical functors with $T_{q_*} \simeq T_{q_*|_{\mathcal{A}}} \circ T_{q_*|_{\mathcal{B}}}$. Since $q_*\mathcal{O}_{\tilde{E}_i}(-1) \simeq \mathcal{O}_{E_i}(-1)$, we see that $T_{q_*|_{\mathcal{A}}} \simeq \prod_i T_{\mathcal{O}_{E_i}(-1)}$ and hence

$$T_{F''} \simeq T_{q_*|_{\mathcal{A}}}^{-1} \circ T_{q_*} \simeq \prod_i T_{\mathcal{O}_{E_i}(-1)}^{-1} \circ M_{\mathcal{O}_K(E/2)}[1].$$

Notice that the cotwist of $F'' \simeq q_*|_{\mathcal{B}}$ is given by $S_A \circ \iota^*[-2] \simeq \iota^*$. \square

Remark 2.2. We can use equation (1) below to rewrite this decomposition as

$$T_{F''} \simeq \prod_i T_{\mathcal{O}_{E_i}} \circ M_{\mathcal{O}_K(-E/2)}[1].$$

Lemma 2.3. We have the following isomorphism of functors

$$F[1] \simeq T_{\mathcal{O}_K} \circ F''.$$

Proof. Consider the following exact triangles of functors

$$\mathrm{Hom}^*(\mathcal{O}_K, F'') \otimes \mathcal{O}_K \rightarrow F'' \rightarrow T_{\mathcal{O}_K} \circ F'' \quad \text{and} \quad F' \rightarrow F'' \rightarrow F[1].$$

Then it is sufficient to show that $\mathrm{Hom}^*(\mathcal{O}_K, F'') \otimes \mathcal{O}_K \simeq F' \simeq H^*(A, _) \otimes \mathcal{O}_K$. In other words, it is enough to show that $H^*(K, F''(_)) \simeq H^*(A, _)$ but this follows from the fact that p is a blowup. Indeed, we have

$$H^*(K, F''(_)) \simeq H^*(K, q_*p^*(_)) \simeq H^*(\tilde{A}, p^*(_)) \simeq H^*(A, p_*p^*(_)) \simeq H^*(A, _). \quad \square$$

Corollary 2.4. $F : \mathcal{D}(A) \rightarrow \mathcal{D}(K)$ is a spherical functor with cotwist $C_F \simeq \iota^*$ and twist

$$T_F \simeq T_{\mathcal{O}_K} \circ T_{F''} \circ T_{\mathcal{O}_K}^{-1}.$$

Proof. Recall that if $F : \mathcal{D}(Z) \rightarrow \mathcal{D}(Y)$ is a spherical functor and $\Phi : \mathcal{D}(Y) \xrightarrow{\sim} \mathcal{D}(X)$ is an equivalence of categories then $\Phi \circ F : \mathcal{D}(Z) \rightarrow \mathcal{D}(X)$ is also a spherical functor with the same cotwist and $T_{\Phi \circ F} \simeq \Phi \circ T_F \circ \Phi^{-1}$. In particular, we see immediately from Lemma 2.3 that F is a spherical functor with cotwist $C_F \simeq \iota^*$ and twist

$$T_F \simeq T_{F[1]} \simeq T_{\mathcal{O}_K} \circ T_{F''} \circ T_{\mathcal{O}_K}^{-1}. \quad \square$$

Corollary 2.5. *The squares of the spherical twists are given by*

$$T_F^2 \simeq T_{F''}^2 \simeq [2].$$

In particular, $T_F^2, T_{F''}^2$ act trivially on cohomology.

Proof. Let $j : E \rightarrow K$ denote the inclusion of the exceptional divisor. Since E is smooth, we can apply [Add11, §1.2, Example 5] to see that $j_* : \mathcal{D}(E) \rightarrow \mathcal{D}(K)$ is spherical with cotwist $C_{j_*} \simeq M_{\mathcal{O}_E(E)}[-1] \simeq S_E[-2]$ and twist $T_{j_*} \simeq M_{\mathcal{O}_K(E)}$.

Set $\mathcal{A}_1 := \langle \mathcal{O}_{E_1}(-1), \dots, \mathcal{O}_{E_{16}}(-1) \rangle$ and $\mathcal{A}_2 := \mathcal{A}_1 \otimes \mathcal{O}_E(1)$ to be subcategories of $\mathcal{D}(E)$. Then, by [Orl92, Theorem 2.6], we have a semi-orthogonal decomposition

$$\mathcal{D}(E) \simeq \langle \mathcal{A}_1, \mathcal{A}_2 \rangle$$

Thus, using Kuznetsov's trick [AA13, Theorem 11] (which is a special case of [HLS13, Theorem 4.13]), we see that the restriction $j_\ell := j_*|_{\mathcal{A}_\ell} : \mathcal{D}(\mathcal{A}_\ell) \rightarrow \mathcal{D}(K)$ is spherical for each $\ell = 1, 2$ and the twists satisfy $T_{j_1} \circ T_{j_2} \simeq T_{j_*}$. That is

$$\prod_i T_{\mathcal{O}_{E_i}(-1)} \circ \prod_i T_{\mathcal{O}_{E_i}} \simeq M_{\mathcal{O}_K(E)}. \quad (1)$$

Furthermore, we have $j_1 \simeq M_{\mathcal{O}_K(E/2)} \circ j_2$ since $\mathcal{O}_{E_i}(E/2) \simeq \mathcal{O}_{E_i}(-1)$ and so

$$T_{j_1} \simeq T_{M_{\mathcal{O}_K(E/2)} \circ j_2} \simeq M_{\mathcal{O}_K(E/2)} \circ T_{j_2} \circ M_{\mathcal{O}_K(-E/2)}$$

which, after taking inverses, equates to

$$\prod_i T_{\mathcal{O}_{E_i}(-1)}^{-1} \circ M_{\mathcal{O}_K(E/2)} \simeq M_{\mathcal{O}_K(E/2)} \circ \prod_i T_{\mathcal{O}_{E_i}}^{-1}. \quad (2)$$

This expression allows us to reduce the formula for $T_{F''}^2$ in the following way:

$$\begin{aligned} T_{F''}^2 &\simeq \prod_i T_{\mathcal{O}_{E_i}(-1)}^{-1} \circ M_{\mathcal{O}_K(E/2)} \circ \prod_i T_{\mathcal{O}_{E_i}(-1)}^{-1} \circ M_{\mathcal{O}_K(E/2)}[2] \\ &\simeq M_{\mathcal{O}_K(E/2)} \circ \prod_i T_{\mathcal{O}_{E_i}}^{-1} \circ \prod_i T_{\mathcal{O}_{E_i}(-1)}^{-1} \circ M_{\mathcal{O}_K(E/2)}[2] \\ &\simeq M_{\mathcal{O}_K(E/2)} \circ M_{\mathcal{O}_K(-E)} \circ M_{\mathcal{O}_K(E/2)}[2] \\ &\simeq [2] \end{aligned}$$

where the second and third lines follow from equations (2) and (1) respectively.

The fact that $T_F^2 \simeq [2]$ now follows immediately from Corollary 2.4. \square

Corollary 2.6. $\mathrm{im} F$ and $\mathrm{im} F''$ are spanning classes for $\mathcal{D}(K)$.

Proof. For any spherical functor $F : \mathcal{D}(Y) \rightarrow \mathcal{D}(X)$, we have a natural spanning class for $\mathcal{D}(X)$ given by $\mathrm{im} F \cup (\mathrm{im} F)^\perp \simeq \mathrm{im} F \cup \ker R$; see [Add11, §1.4]. However, in our case we have $\ker R = 0$. Indeed, let $\mathcal{E} \in \ker R$. Then the defining triangle for the twist $FR(\mathcal{E}) \rightarrow \mathcal{E} \rightarrow T_F(\mathcal{E})$ shows that $T_F(\mathcal{E}) \simeq \mathcal{E}$. But by Corollary 2.5, we have $\mathcal{E} \simeq T_F^2(\mathcal{E}) \simeq \mathcal{E}[2]$ which implies $\mathcal{E} \simeq 0$; a similar argument works for F'' . \square

Remark 2.7. This should be contrasted to the object case where every spherical object \mathcal{E} is expected to have a non-empty perpendicular \mathcal{E}^\perp ; [Plo05, Question 1.25].

Lemma 2.8. *The functors $F, F'' : \mathcal{D}(A) \rightarrow \mathcal{D}(K)$ are actually split spherical. That is, the natural triangles associated to the units η, η'' of adjunction are split. In particular, this implies that F and F'' are faithful.*

Proof. We prove the statement only for F since F'' is identical. In order to show that the triangle $\mathrm{id}_A \xrightarrow{\eta} RF \rightarrow \iota^*$ is split, it suffices to show that $\mathrm{Ext}^1(\mathrm{id}_A, \iota^*) = 0$. But on the level of kernels, this is just

$$\begin{aligned} \mathrm{Ext}_{A \times A}^1(\Delta_* \mathcal{O}_A, \mathcal{O}_{\Gamma_\iota}) &\simeq \mathrm{Ext}_A^1(\mathcal{O}_A, \Delta^! \mathcal{O}_{\Gamma_\iota}) \quad \text{by adjunction} \\ &\simeq \mathrm{Ext}_A^1(\mathcal{O}_A, \Delta^* \mathcal{O}_{\Gamma_\iota}[-2]) \\ &\simeq H^{-1}(A, \mathcal{O}_{A[2]}) = 0. \end{aligned} \quad \square$$

Proposition 2.9. *The induced map on cohomology $F^H : H^*(A, \mathbb{Q}) \rightarrow H^*(K, \mathbb{Q})$ is injective on $H^{\mathrm{even}}(A, \mathbb{Q})$, zero on $H^{\mathrm{odd}}(A, \mathbb{Q})$ and the twist T_F acts on $H^*(K, \mathbb{Q})$ by reflection in $(\mathrm{im} F^H)^\perp$ with respect to the Mukai pairing.*

Proof. The first statement follows from the fact that $R^H F^H \simeq \mathrm{id}_{H^*(A, \mathbb{Q})} + \iota^{*H}$ and ι^{*H} acts by the identity on $H^{\mathrm{even}}(A, \mathbb{Q})$ and by -1 on $H^{\mathrm{odd}}(A, \mathbb{Q})$. Next, the defining triangle for the twist gives $T_F^H \simeq \mathrm{id}_{H^*(K, \mathbb{Q})} - F^H R^H$ from which it follows immediately that everything in $\ker R^H \simeq (\mathrm{im} F^H)^\perp$ is fixed by T_F^H . Finally, to see that T_F^H acts on $\mathrm{im} F^H$ as -1 we observe that $T_F \circ F \simeq F \circ C_F[1] \simeq F \circ \iota^*[1] \simeq F[1]$ and so the claim follows. \square

Remark 2.10. Notice that this is very different to the object case where the twist acts on cohomology by reflection in a *hyperplane*; see [Huy06, Corollary 8.13] for more details. It follows from Proposition 2.9 that our twist is acting on cohomology by reflection in a subspace of codimension $8 = \dim H^{\mathrm{even}}(A, \mathbb{Q})$.

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